Acceleration Beyond the Wave Speed in Dissipative Wave-Particle Systems

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Abstract

When an isotropic object is accelerated by plane waves, its limiting speed is less or equal to the wave speed. We study the acceleration of anisotropic objects in classical wave-particle systems. We investigate objects with anisotropic mass distributions subject to anisotropic friction forces. We find that anisotropic objects can be trapped by the waves, and reach a limiting speed that is larger than the wave speed. We discuss particle accelerator applications.

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Waves can be used to both accelerate objects to very high velocities and to slow them down to very low velocities. Recently, Kitagawa et al. [1] showed that a 10GV/m electric field in ultraintense-laser-illuminated capillaries can accelerate plasma electrons to a speed, which corresponds to a kinetic energy of 100 MeV. Mourou et al. [2, 3] proposed a chirped-pulse amplification method for ultrahigh-power short laser pulses ($\leq$ 1 ps, $\geq$ 10 TW). With this setup even tabletop lasers might be able to produce such large fields. Hoshino, Shimada [4], and Vasiliev [5] have shown that electrons are likely to be trapped by large amplitude electrostatic shock waves. During the trapping phase they then can be effectively accelerated by the shock motional electric field. This method is related to earlier relativistic particle acceleration techniques, where plasma surface waves, laser radiation pressure, and laser-plasma interactions are used to accelerate particles to arbitrarily high energies [6–9]. However, these early wake field accelerators and surfatron models are inefficient, because the electrons tend to detrap[10]. Tajima and Dawson [11] showed for that the problem with detrapping can be overcome by imposing a magnetic field of appropriate strength perpendicular to the plasma wave[12, 13]. This deflects particles parallel to the phase fronts of the accelerating wave, causing them to move in phase with the wave. Another approach to ensure trapping was suggested by Shvets and Fisch[14]. They conceived a hybrid of laser-wakefield and plasma-wakefield accelerators, in which an intense laser pulse is optically guided by a plasma channel, created by the leading portion of a high-current, low-energy electron beam.

In the low-velocity range, is the most active research fields in the area of wave particle acceleration/deceleration are laser cooling of trapped particles, first proposed by Wineland and Dehmelt in 1975 [15], and acceleration and trapping of particles by radiation pressure [16–18]. The anisotropic interaction of the electric field vector of intense laser radiation with the dipole moment can be utilized for both laser alignment and spatial trapping of molecules [19].

In contrast to the low-velocity wave acceleration techniques, all previous work on high-velocity wave acceleration techniques has focused on point-like isotropic objects with waves and time dependent fields. In this paper we investigate the high-velocity acceleration of anisotropic, classical objects with a small, but finite size. We find that objects, whose viscous friction force depends on their orientation, can be accelerated beyond the wave speed by planar sinusoidal waves. This means that anisotropic objects can be trapped by the wave if the wave amplitude exceeds a critical value. This phenomenon is attributed the fact that their limiting velocity has a non-zero component perpendicular the wave vector. We show that the limiting speed reaches a maximum, for a certain orientation of the object. Additionally, we show that objects with a certain variation of the friction constant, along their principal axis, approach the optimal orientation automatically. We discuss potential accelerator applications.

We consider an object with two masses m_r and m_f located at $\vec{x}_r$ and $\vec{x}_f = \vec{x}_r + L\vec{d}$. The distance L between the two masses is constant. $d = (cos\alpha, sin\alpha, 0)$ is the direction of principal axis of the object. The object is accelerated by a plane wave with wave vector, $\vec{k} = (k, 0, 0)$, amplitude A and wave speed c, where the force $F_f\vec{k} = \kappa_f A\cos(\vec{k}\vec{x}_f - kct)\vec{k}$ acts on m_f, and $F_r\vec{k} = \kappa_r A\cos(\vec{k}\vec{x}_r - kct)\vec{k}$ acts on m_r, both in the direction of the wave vector $\vec{k}$. The equation of motion for the center of mass $\vec{x}$ is

$$m\ddot{\vec{x}} = -\mu_{\|}\vec{v}_{\|} - \mu_{\perp}\vec{v}_{\perp} + F_f\vec{k} + F_r\vec{k} \tag{1}$$

where $m = m_f + m_r$ is the mass of the object, $\vec{v_{\|}} = (\dot{\vec{x}} \cdot \vec{d})\vec{d}$ is the velocity of the object along it's principle axis. $\vec{v}_{\perp} = (\dot{\vec{x}} \cdot \vec{n})\vec{n}$ is the velocity perpendicular to the principal axis, where $\vec{n} = (-\sin\alpha, \cos\alpha, 0)$ is the corresponding normal vector. The net friction coefficients are $\mu_{\|} = \mu_{\|f} + \mu_{\|r}$ and $\mu_{\perp} = \mu_{\perp f} + \mu_{\perp r}$, where $\mu_{\|f}$ and $\mu_{\|r}$ are the friction coefficients along the principal axis for both particles and $\mu_{\perp f}$ and $\mu_{\perp r}$ are the friction coefficients perpendicular to the principal axis. For an object floating on a fluid with shallow surface waves, the coupling constants are $\kappa_f = m_f g$ and $\kappa_r = m_r g$, where g is the gravitational constant.

The equation of motion for the rotation about the center of mass of the object is

$$\begin{aligned} J\ddot{\vec{\alpha}} &= \mu\dot{\vec{\alpha}} + L_f\vec{d} \times \left(-\mu_{\perp f}\vec{v}_{\perp} + F_f\vec{k}\right) \\ &\quad -L_r\vec{d} \times \left(-\mu_{\perp r}\vec{v}_{\perp} + F_r\vec{k}\right) \end{aligned} \tag{2}$$

where $\vec{\alpha} = (0, 0, \alpha)$, $\mu = \mu_{\perp f}L_f^2 + \mu_{\perp r}L_r^2$, $L_f = Lm_r/m$, and $L_r = Lm_f/m$. $J = m_f L_f^2 + m_r L_r^2$ is the interia. In the following we assume that the length L is small compared to the wave length, $L \ll 2\pi/k$ and use the following approximation $\vec{F}_f + \vec{F}_r \approx (\kappa_f + \kappa_r)A\cos(\vec{x}\vec{k} - kct)$.

Numerical results generated from Eqs. (1) and (2) with a Runge-Kutta algorithm (5^{th}–6^{th} order, time step $\Delta t = 0.1$) are shown in Fig. 1 and Fig. 2. Fig. 1 shows a system where the x-velocity v_x approaches the wave speed c, and the y-velocity v_y approaches a constant non-zero limiting value. The particle motion is found to be trapped by the wave. The figure indicates that the limiting speed of the object is larger than the wave speed, i.e. $v_\infty = \sqrt{v_{x,\infty}^2 + v_{y,\infty}^2} > c$, where $v_{x,\infty} = \dot{x}(t=\infty)$ and $v_{y,\infty} = \dot{y}(t=\infty)$. In contrast, Fig. 2 illustrates the dynamics of a system where the wave amplitude, A, which is much smaller. Here the limiting y-velocity is zero. Further, the limiting value of the average x-velocity is less than c, hence the particle is not trapped by the wave. Therefore, the limiting speed of the object is less than the wave speed, i.e. $v_\infty < c$.

If the particle is trapped by the wave, the limiting x-velocity is the wave speed: $v_{x,\infty} = c$. The x-position is $x = x_w + \Delta x$; where the distance Δx to the nearest wave front, located at $x_w = ct + n * (2\pi/k)$, approaches a constant value Δx_∞. The integer n depends on the initial conditions. The limiting distance between the nearest wavefront and the particle is:

$$\Delta x_\infty = \frac{1}{k} \arccos \frac{\mu_\parallel c}{\kappa A k(\cos^2 \alpha_\infty + r_\mu \sin^2 \alpha_\infty)} \tag{3}$$

where $\kappa = \kappa_f + \kappa_r$. $\alpha_\infty = \alpha(t=\infty)$ is the limiting orientation of the object and $r_\mu = \mu_\parallel/\mu_\perp$ is the friction anisotropy.

The argument of the inverse cosine-function $F = \mu_\parallel c/(\kappa A k(\cos^2 \alpha_\infty + r_\mu \sin^2 \alpha_\infty))$ in Eq. (3) is the ratio between the x-component of the friction force and the force on the object by the wave. If $F > 1$ the object de-traps. F depends on the limiting orientation α_∞. If α_∞ exceeds a critical angle, the object de-traps. The critical α-value is

$$\alpha_c = \begin{cases} 0 & \text{if } A < \frac{\mu_\parallel c}{\kappa k} \text{ (always detrapped)} \\ \pi/2 & \text{if } A > \frac{\mu_\parallel c}{\kappa k r_\mu} \text{ (always trapped)} \\ \arcsin\sqrt{f} & \text{else (trapped if } |\alpha_\infty| \mod 2\pi < \alpha_c) \end{cases} \tag{4}$$

where $f = (1 - \frac{\mu_\parallel c}{\kappa A k})/(1 - r_\mu)$. The limiting y-velocity for the trapped object $v_{y,\infty}$ is dependent on the limiting orientation, α_∞, and the friction anisotropy, r_μ.

$$v_{y,\infty} = c\frac{(1 - r_\mu)\sin\alpha_\infty \cos\alpha_\infty}{cos^2\alpha_\infty + r_\mu \sin^2 \alpha_\infty} \tag{5}$$

Fig. 3 shows the limiting y-velocity versus the orientation of the object. For a given r_μ the limiting y-velocity reaches a maximum if the limiting orientation α_∞ has the value

$$\alpha_m = \pm(\arccos\sqrt{\frac{r_\mu}{1 + r_\mu}} + i * 2\pi) \tag{6}$$

where $i = 0, 1, 2, \ldots$. The corresponding maximum limiting y-velocity is $v_y(\alpha_m) = c(1 - r_\mu)/(2\sqrt{r_\mu})$. However the object may de-trap at the optimal α-value. Hence, the maximum achievable limiting y-velocity V_y for an object with friction anisotropy r_μ is

$$V_y = \begin{cases} v_{y,\infty}(\alpha_m) & \text{if } \alpha_m(r_\mu) < \alpha_c(r_\mu) \\ v_{y,\infty}(\alpha_c) & \text{else} \end{cases} \tag{7}$$

Fig. 4 shows the maximum achievable limiting y-velocity V_y versus the the friction anisotropy r_μ. For a small object ($L \ll 2\pi/k$) with coupling $\kappa_f = m_f g$, $\kappa_r = m_r g$, the object can reach V_y, if the variation of the friction along the principal axis, $R = \mu_{\perp f}/\mu_\perp$, has the following value:

$$R = \frac{m_f}{m}\left(1 + \frac{m_r g A k^2 L}{\mu_\perp v_{\perp,\infty}} \sin\left(k\Delta x_\infty\right) \sin\left(2\alpha_\infty\right)\right) \tag{8}$$

where $\alpha_\infty = \alpha_m$ or $\alpha_\infty = \alpha_c$ as indicated in Eq. (7). $v_{\perp,\infty} = v_{y,\infty}\cos\alpha_\infty - c\sin\alpha_\infty$ is component of the limiting velocity in the direction of $\vec{n}$. Δx_∞ is given by Eq. (3). An object with the above R-value is accelerated to the theoretical limit of the y-velocity and reaches the largest speed possible in this system. Fig. 5 shows that trapping occurs at roughly the same wave amplitude for all initial orientations.

We have shown that classical anisotropic objects, subject to viscous friction, can be accelerated with planar sinusoidal waves beyond the wave speed. When the object is trapped, the component of limiting speed in the direction of the wave vector, $v_{x\infty}$ equals the phase velocity of the wave. Hence, we expect that for wave packets where the phase velocity exceeds the wave speed [20], even $v_{x,\infty}$ can exceed the wave speed. In this paper we studied classical objects. It should be possible to generalize these results to relativistic particles in a medium, i.e. accelerate anisotropic objects with light waves beyond the speed of light in the medium. The relativistic wave-particle system differs in two ways, (i) the leading dissipation mechanism is Cerenkov radiation [21, 22, 22–26] and (ii) the mass and energy of the object diverge[27] at the speed of light in vacuum. We expect that laser-plasma beat-wave accelerator can trap certain anisotropic particles even without a large perpendicular magnetic field [11–13].

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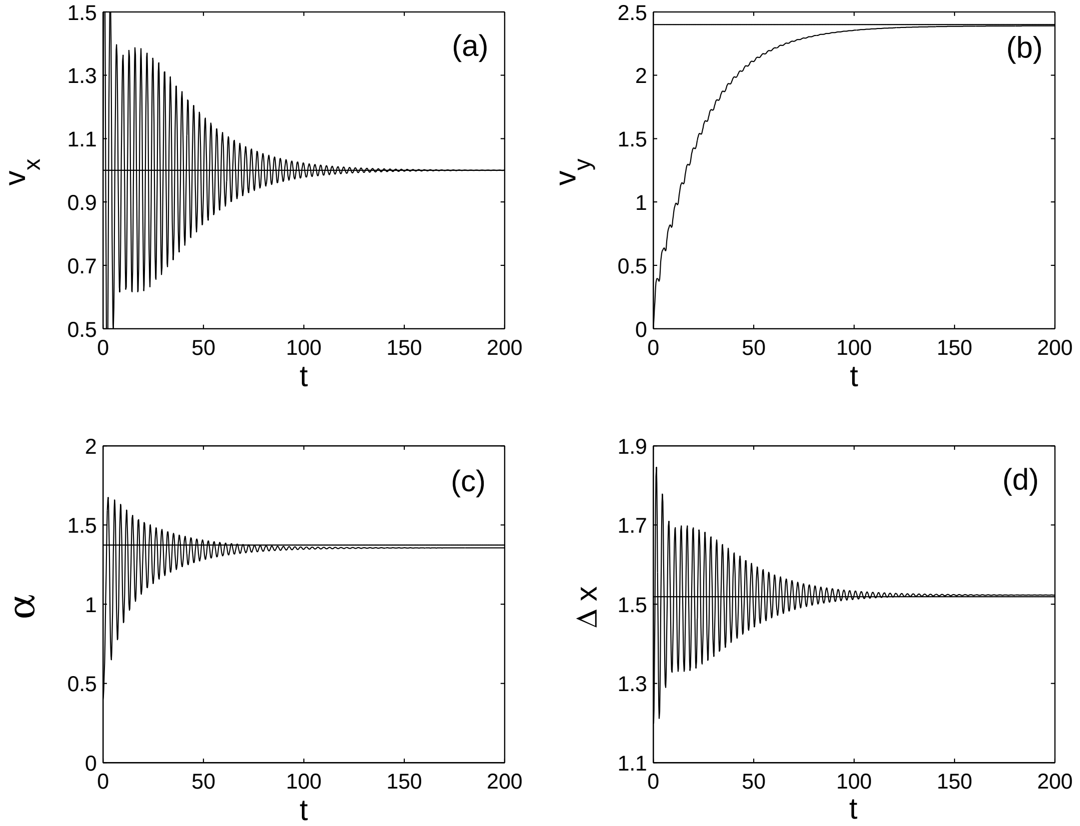

FIG. 1: Numerical simulation of a wave-particle dissipative system for the trapped case with theoretical projected values superimposed as dotted lines: x velocity, v_x, with respect to time (a); y velocity, v_y, with respect to time (b); α with respect to time (c); and the distance to the nearest wave front Δx with respect to time (d). All parameters are equal to unity except for $\mu_{||} = 0.04$, $\mu_{\perp f} = 0.41$, $\mu_{\perp r} = 0.59$, and $L_f = L_r = 0.05$. The horizontal lines indicate the theoretical values for the limiting state of a trapped object according to Eq. 3, Eq. 5, and Eq. 8.

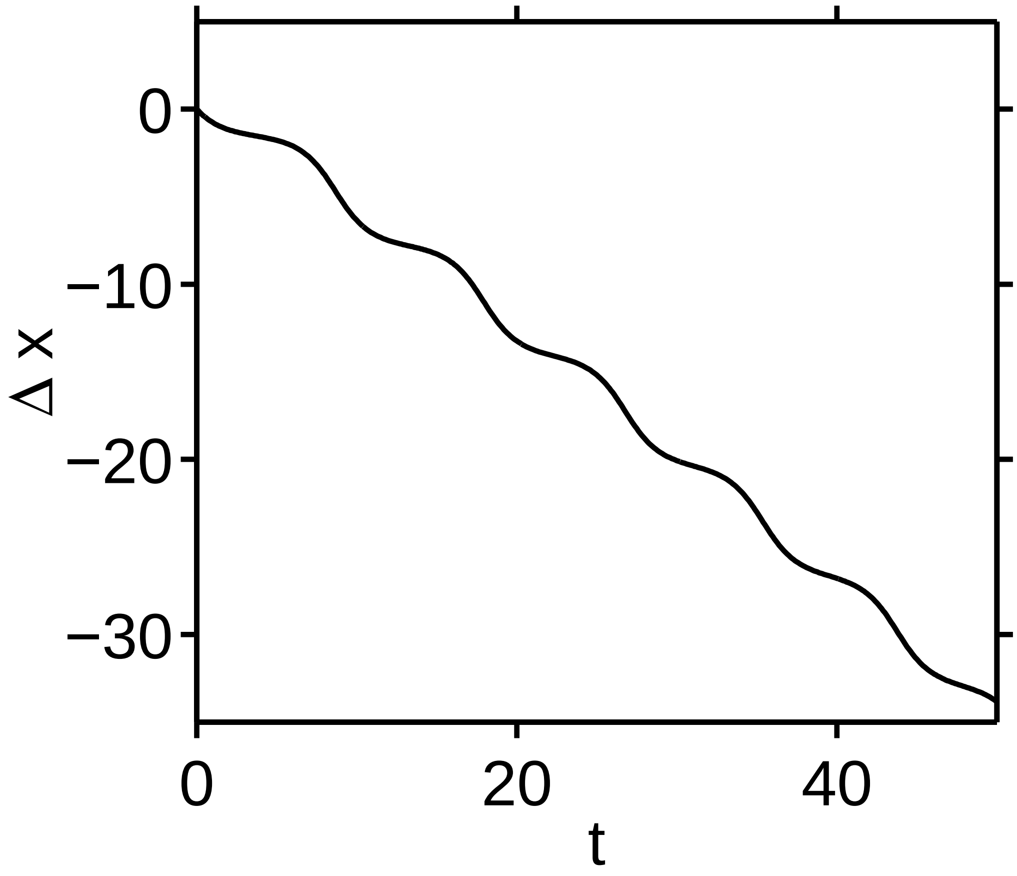

FIG. 2: Numerical simulation of Δx versus time when the object is de-trapped. The parameters are the same as in Fig. 1 except for $A = 0.5$.

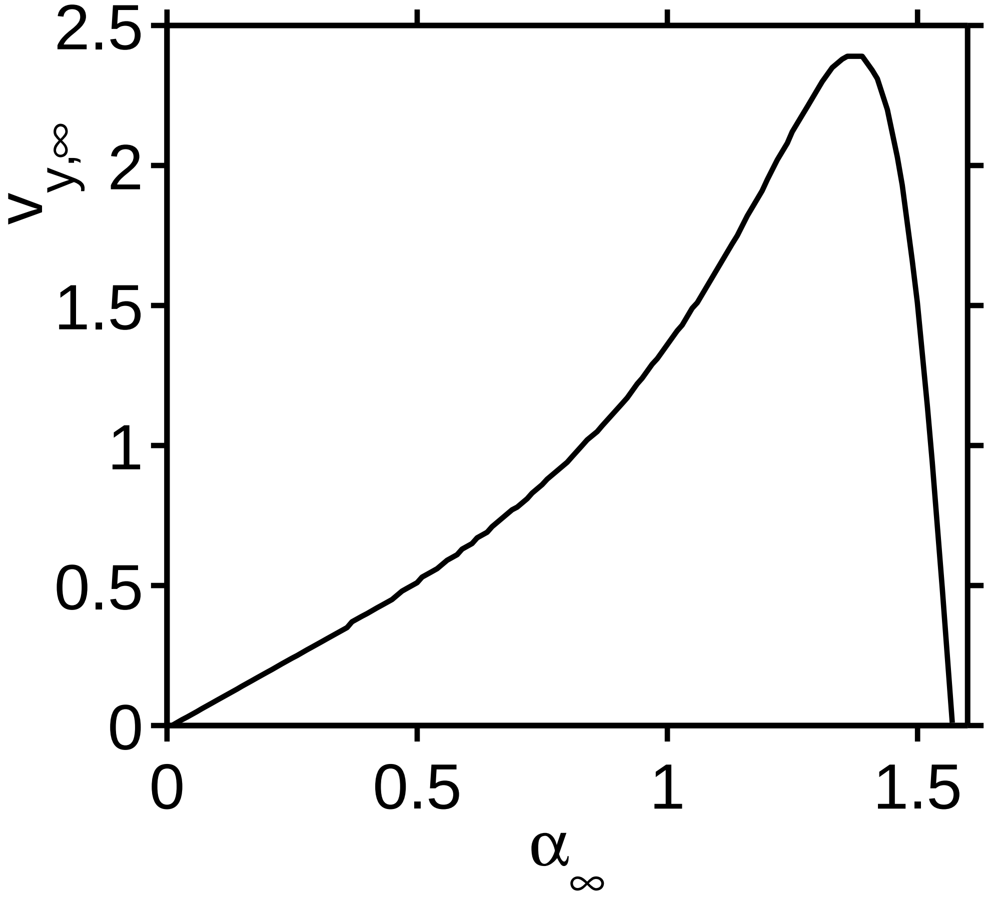

FIG. 3: The limiting y-velocity $v_{y,\infty}$ as a function of the limiting orientation α_∞ for friction anisotropy $r_\mu = 0.04$

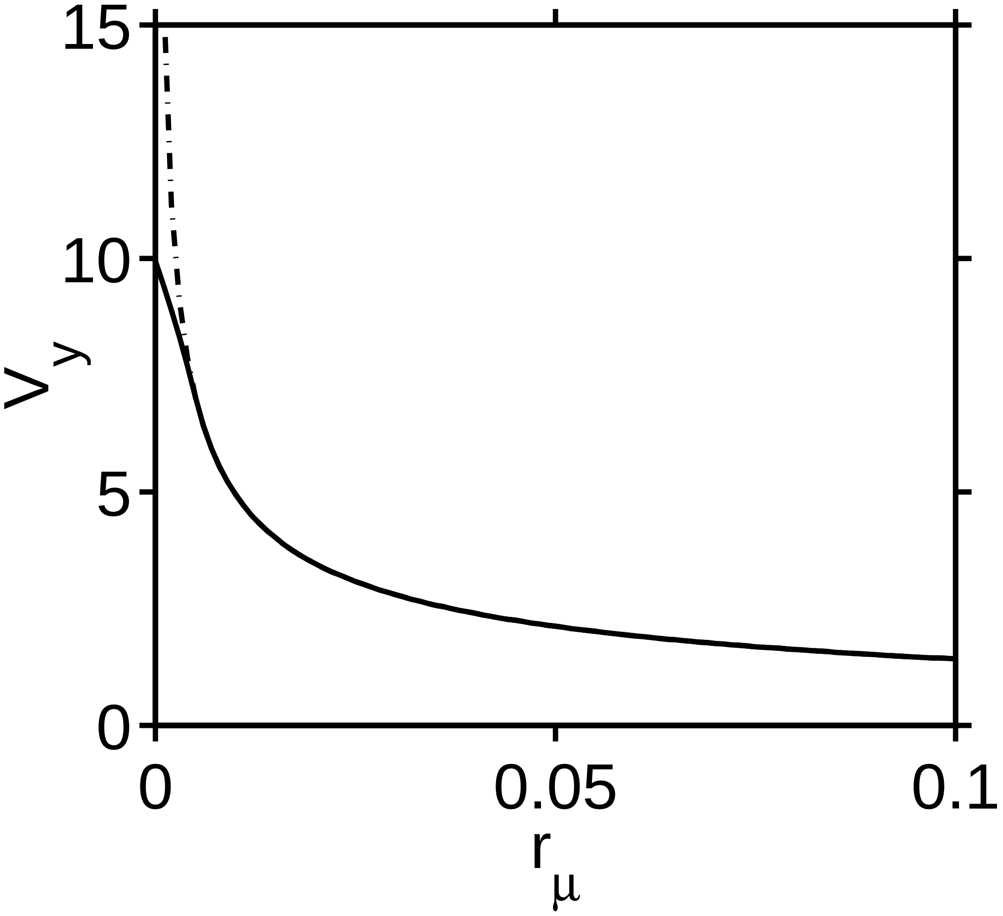

FIG. 4: The maximum achievable limiting y-velocity V_y as a function of the friction anisotropy r_μ. The dashed line shows $v_y(\alpha_m)$. All parameters are the same as in Fig. 1 except for $\mu_\parallel = 0.1$.

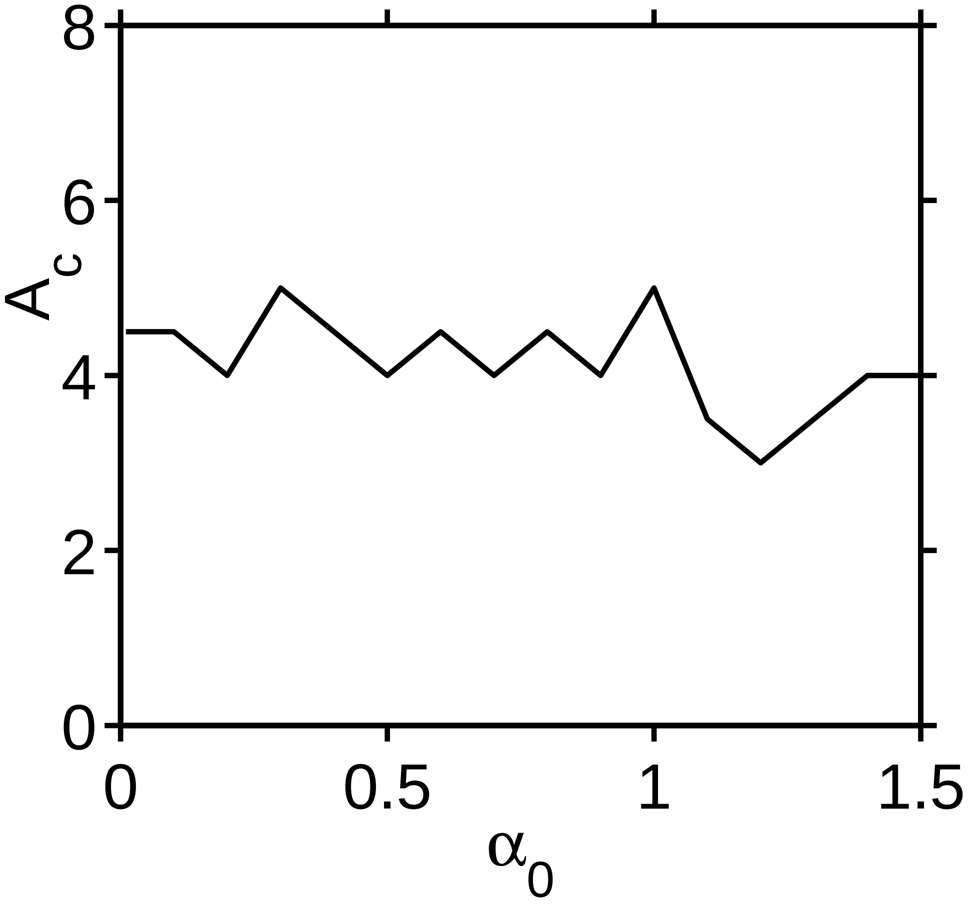

FIG. 5: The minimum wave amplitude A_c to trap the object as a function of the initial orientation of the object α_0. The parameters are the same as in Fig. 1 except for the wave amplitude.